

Chapter 2 Solved Exercises

Question 1. (Similar to Exercise 2.1) Express $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, \dots , $\frac{9}{10}$ as Egyptian fractions. Do not use the same part more than once (for example, you should not write $\frac{3}{10}$ as $\frac{1}{10} + \frac{1}{10} + \frac{1}{10}$) and try to use as few parts as possible. Write your results in both Egyptian and modern notation.

Question 2. (Exercise 2.2) The prevalence of computers has led to increasingly common usage of decimals to express fractional parts. Give two advantages and two disadvantages to writing fractions in decimal form.

Question 3. (Similar to Exercise 2.6) Problem 31 of the *Rhind papyrus* asks this:

A quantity, its $\frac{2}{3}$, its $\frac{1}{2}$, and its $\frac{1}{7}$, added together, become 33. What is this quantity?

Solve this problem by making an incorrect guess, noting what proportion your answer is of 33, and then scaling your guess appropriately.

Question 4. (Similar to Exercise 2.10(a) and (b)) Ancient Egyptians also had a method to divide two integers, which is similar to their multiplication approach. For example, if they wanted to determine $147 \div 7$, they would instead ask what one would have to multiply 7 by to reach 147.

(a) With this background, explain why the below procedure successfully computes $147 \div 7$.

$$\begin{array}{r}
 \textcircled{1 \rightarrow 7} \\
 2 \rightarrow 14 \\
 \textcircled{4 \rightarrow 28} \\
 8 \rightarrow 56 \\
 \textcircled{16 \rightarrow 112}
 \end{array}
 \begin{array}{r}
 \diagdown \\
 \diagdown \\
 \diagdown \\
 \diagup \\
 \diagup \\
 \diagup
 \end{array}
 \begin{array}{r}
 1 \\
 + 4 \\
 \hline
 16 \\
 \hline
 21
 \end{array}$$

(b) Create another division example and show how the above approach solves it. Pick numbers for which the quotient is an integer.

Question 5. (Similar to Exercise 2.13) Read Shout-Out 2.1. Then, iterate that process two more times to find larger squares and closer approximations to $\sqrt{2}$. For each, write down, as a decimal, how far off your rational approximation is from $\sqrt{2}$.

Question 6. (Similar to Exercise 2.14) Use Hero's method to find an estimate of $\sqrt{5}$ that is within 10^{-10} of the exact answer.

Question 7. (Similar to Exercise 2.15 (a)) This chapter discussed “excess and deficit” problems. Solve the following by making two incorrect guesses, and then using the techniques we discussed to solve the problem.

This problem comes from the ancient Chinese text *The Nine Chapters on the Mathematical Art*.

A collection of chickens are purchased jointly among a group of people. If everyone contributes 9 coins, the excess is 11 coins, and if everyone contributes 6 coins, the deficit is 16 coins. Find the number of people and the total cost of the collection of chickens.

Question 8. (Exercise 2.22) Write an essay on Indian mathematician Mahāvīra, including his work on Egyptian fractions.

Question 9. (Exercise 2.24) Write a 300-word essay on the Egyptian Mathematical Leather Roll.

Question 10. (Exercise 2.28) Come up with your own exercise that uses the information from this chapter. Make sure it is different from the other exercises in this chapter and have its level of difficulty be at about the same level as the others. Then, answer your question.