Chapter 2 Solved Exercises

- You must prove all of your answers (unless stated otherwise).
- Remember the proof techniques that you learned in your intro-to-proofs course: Direct, by contradiction, by using the contrapositive, by induction, and/or by cases. And you can disprove something by exhibiting a counterexample.

Question 1. (Similar to Exercise 2.1)

(a) List all the elements of $\mathcal{P}(\{a, ☺, 7\})$.

(b) List all the elements of $\mathcal{P}(\{y\})$.

(c) Determine a formula for the number of elements in the power set of an n-element set. You do not need to prove that your formula works.

Question 2. (Similar Exercise 1.3) The following pairs of sets have the same size, and so there exists a bijection between them. Write down an explicit bijection in each case. You do not need to prove your answers.

(a) $(5, \infty)$ and $(11, \infty)$

(b) $(5, \infty)$ and $(-\infty, 11)$

(c) $(0, \infty)$ and $(0, 1)$

(d) $\mathbb{R}$ and $(0, \infty)$

Question 3. (Exercise 2.16) Show that the smallest infinity is $|\mathbb{N}|$. That is, show that if, $A \subseteq \mathbb{N}$ then either A is finite or $|A| = |\mathbb{N}|$.

Question 4. (Exercise 2.5(a)) Suppose that A and B are countable infinite sets. Prove that $A \cup B$ is also a countable set.

Question 5. (Exercise 2.6) Show that $|\mathbb{N}| = |\mathbb{Z}|$ by finding an explicit bijection from $\mathbb{N}$ to $\mathbb{Z}$. You do not need to prove your bijection works.

Question 6. (Exercise 2.13) Show that there are uncountably many irrational numbers.

Question 7. (Covered in the closing pages of Chapter 1) Is the set of all rational numbers $\frac{m}{n}$ where $|n| \leq 10$ dense in $\mathbb{R}$?

Question 8. State the *Bijection principle.*

Question 9. Define what it means for an infinite set to be *countable* or *uncountable.*

Question 10. State the *continuum hypothesis.*