Chapter 1 Solutions to Selected Exercises

Notes:

- The questions are in a separate PDF on LongFormMath.com.
- Some problems may have many correct solutions, so the below are not the only correct ways to solve the problems.
- If you spot an error, please email it to me at LongFormMath@gmail.com. Thanks!

Solution to Question 1.

$$\begin{aligned}(1030)_4 &= 1 \cdot 4^3 + 0 \cdot 4^2 + 3 \cdot 4 + 0 \cdot 1 \\ &= 64 + 12 \\ &= 76\end{aligned}$$

$$\begin{aligned}61 &= 2 \cdot 27 + 0 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 \\ &= 2 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3 + 1 \cdot 1 \\ &= (2021)_3.\end{aligned}$$

Solution to Question 2.

(a) ♡♡★☺☺☺= 16 + 16 + 4 + 1 + 1 + 1 = 39.

(b) ♡★@♡♡= 64 + 16 + 16 + 16 + 4 = 116.

(c) 91= 64 + 16 + 4 + 4 + 1+1+1 = @♡★★☺☺☺

(d) 180 = 64 + 64 + 16 + 16 + 16 + 4 = @@ ♡♡♡★

Solution of Question 3.

Chinese	Egyptian	Roman	Hindu-Arabic	Base-6
二千一百二十三	𓆼𓍢𓎆𓏺𓏺 𓏺𓏺𓏺	MMCXXIII	2123	$(13455)_6$
五千四百六	𓆼𓆼𓆼𓆼𓆼𓎆𓎆𓎆𓎆 𓏺𓏺𓏺𓏺𓏺𓏺	MMMMMCCCCVI	5406	$(41010)_6$
一百三十七	𓎆𓏺𓏺𓏺 𓏺𓏺𓏺𓏺𓏺𓏺𓏺	CXXXVII	137	$(345)_6$
八百一十三	𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺 𓏺𓏺𓏺	DCCCXIII	813	$(3433)_6$
四千六十一	𓆼𓆼𓆼𓆼𓏺𓏺𓏺𓏺𓏺𓏺 𓏺	MMMMLXI	4061	$(30445)_6$

Solution of Question 4.

(a) $92 = 2 \cdot 36 + 20 \cdot 1 = (2K)_{36}$

(b) $536 = 14 \cdot 36 + 32 \cdot 1 = (EW)_{36}$

(c) $(SUP)_{36} = 28 \cdot 36^2 + 30 \cdot 36 + 25 \cdot 1 = 37,393$

(d) Answers vary.

□

Solution to Question 5.

(a) There are infinitely many possibilities. Here are four:

 $= 60 + 21 = 81.$

 $= 1 \cdot 60^2 + 21 = 3621.$

 $= 60^3 + 21 = 216,021.$

 $= 60^3 + 0 \cdot 60^2 + 21 \cdot 60 + 0 \cdot 1 = 217,260.$

□

Solution to Question 6.

(a) $60\frac{1}{2} = 60 + 30 \cdot \frac{1}{60} = 1;30$

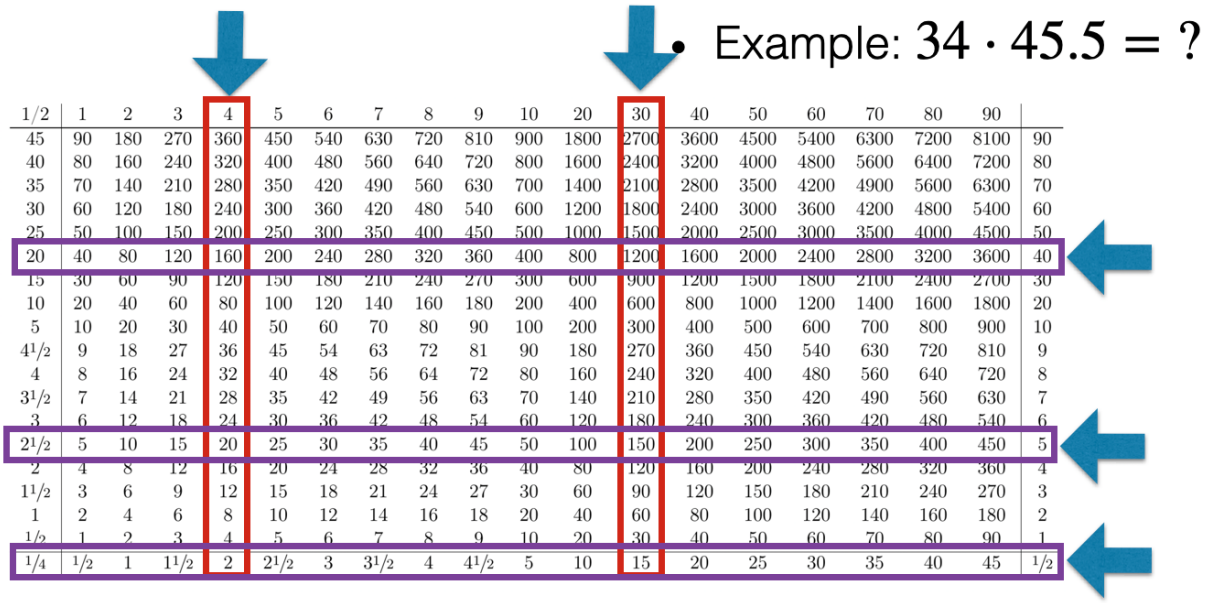
(b) $\frac{1}{90} = \frac{40}{3600} = 0 + 0 \cdot \frac{1}{60} + 40 \cdot \frac{1}{60^2} = 0;0\ 40$

□

Solution to Question 7. Answers vary (with probability 1).

□

Solution to Question 8.



1/2	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	
45	90	180	270	360	450	540	630	720	810	900	1800	2700	3600	4500	5400	6300	7200	8100	90
40	80	160	240	320	400	480	560	640	720	800	1600	2400	3200	4000	4800	5600	6400	7200	80
35	70	140	210	280	350	420	490	560	630	700	1400	2100	2800	3500	4200	4900	5600	6300	70
30	60	120	180	240	300	360	420	480	540	600	1200	1800	2400	3000	3600	4200	4800	5400	60
25	50	100	150	200	250	300	350	400	450	500	1000	1500	2000	2500	3000	3500	4000	4500	50
20	40	80	120	160	200	240	280	320	360	400	800	1200	1600	2000	2400	2800	3200	3600	40
15	30	60	90	120	150	180	210	240	270	300	600	900	1200	1500	1800	2100	2400	2700	30
10	20	40	60	80	100	120	140	160	180	200	400	600	800	1000	1200	1400	1600	1800	20
5	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	10
4½	9	18	27	36	45	54	63	72	81	90	180	270	360	450	540	630	720	810	9
4	8	16	24	32	40	48	56	64	72	80	160	240	320	400	480	560	640	720	8
3½	7	14	21	28	35	42	49	56	63	70	140	210	280	350	420	490	560	630	7
3	6	12	18	24	30	36	42	48	54	60	120	180	240	300	360	420	480	540	6
2½	5	10	15	20	25	30	35	40	45	50	100	150	200	250	300	350	400	450	5
2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	4
1½	3	6	9	12	15	18	21	24	27	30	60	90	120	150	180	210	240	270	3
1	2	4	6	8	10	12	14	16	18	20	40	60	80	100	120	140	160	180	2
½	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	1
¼	½	1	1½	2	2½	3	3½	4	4½	5	10	15	20	25	30	35	40	45	½

$$\begin{aligned} 34 \cdot 45.5 &= (30+4) \cdot (40+5+0.5) \\ &= 30 \cdot 40 + 30 \cdot 5 + 30 \cdot 0.5 + 4 \cdot 40 + 4 \cdot 5 + 4 \cdot 0.5 \\ &= 1547 \end{aligned}$$

□

Solution to Question 9.

Part (a). The total sum of the numbers in the table is

$$1 + 2 + 3 + \cdots + n^2,$$

There is a classic formula for the sum of the first k numbers. This is

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2},$$

which can be proven by induction (or many other ways). Using this formula,

$$1 + 2 + 3 + \cdots + n^2 = \frac{n^2(n^2+1)}{2}.$$

Since there are n columns in this table and they must all have the same sum, each column (and hence each row and diagonal) must sum to

$$\frac{1}{n} \cdot \frac{n^2(n^2+1)}{2} = \frac{n(n^2+1)}{2}.$$

So this is the common sum of each column, row and diagonal. □

Part (b).

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

Solution to Question 10. Answers vary.

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